ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

Μεταπτυχιακό πρόγραμμα ειδίκευσης και συμπληρωματικής ειδίκευσης στη Στατιστική

5 Φεβρουαρίου 2001

ΕΞΕΤΑΣΕΙΣ

Μάθημα : "Προχωρημένες χρονολογικές σειρές"
Διδάσκοντες: Ε. Ιωαννίδης
Χρόνος: 3 ώρες

1st Question

(0.6 points) Assume you observe $X_1,\ldots,X_T$ from the following model:

$$X_t = C_1 \exp\left(it\frac{\pi}{3}\right) + \overline{C}_1 \exp\left(-it\frac{\pi}{3}\right) + C_2 \exp\left(it\frac{2\pi}{3}\right) + \overline{C}_2 \exp\left(-it\frac{2\pi}{3}\right)$$

where C_1 and C_2 are complex valued independent zero-mean random variables with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 4$ respectively. From a preliminary analysis of the data you concluded that $C_1 = 1.2 - i\,0.5$ and $C_2 = -3.6 + i\,2.1$.

Give, in a rough drawing,

a) the spectral distribution function
b) the real and imaginary part of the spectral representation of the specific path $X_1,\ldots,X_T$

2nd Question

a) **(0.2 points)** Assume that the auto-covariances c_u of a stationary process A converge to 0 as u tends to infinity at a faster rate than the auto-covariances of a stationary process B. How do the spectra of the two processes compare?

b) **(0.2 points)** Assume that the auto-covariances of a stationary process do not converge to 0 as u tends to infinity. What would you suggest about the spectral distribution function of the process?

c) **(0.2 points)**Assume that the auto-covariances c_u of a stationary ARMA process A alternate regularly above and below 0 as u increases faster than those of a stationary ARMA process B (which as well alternate regularly above and below 0 as u increases), while they both tend to 0 in magnitude. How do the spectra of the two processes compare?

3rd Question

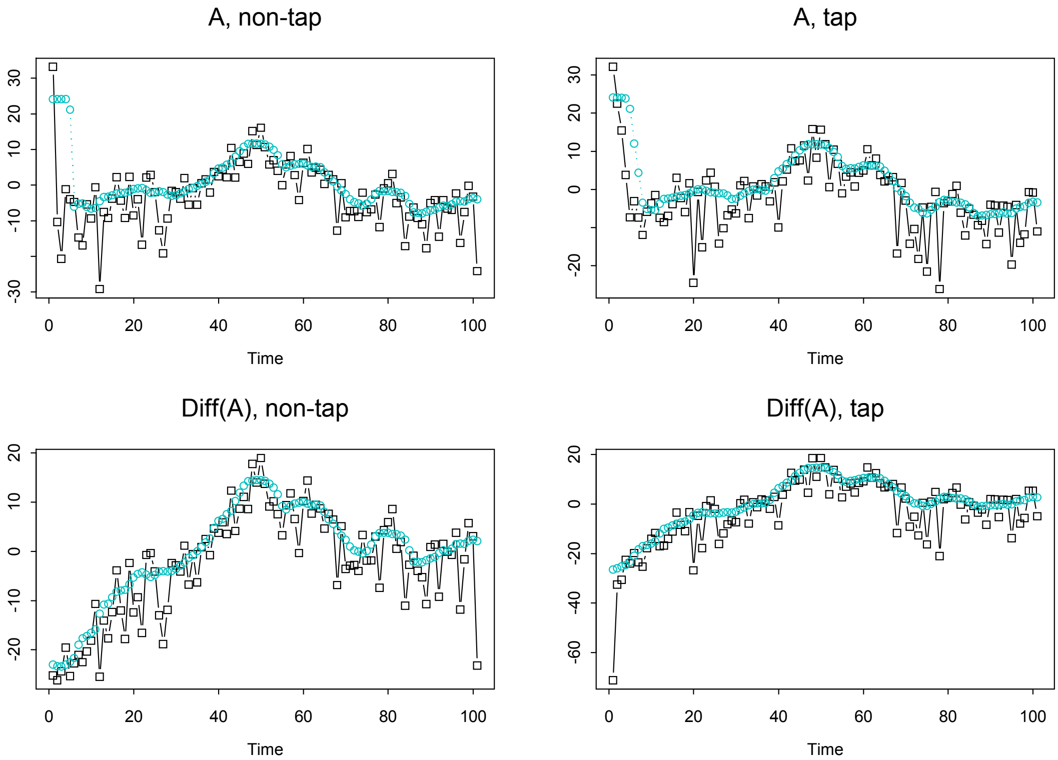
(1.8 points) Below (see also next page) you find 3 sets of figures. Each set is based on 3 different sets of data $X_1,\ldots,X_{200}$ (series A, series B, series C) which are generated from 3 different models. For each set of data you find (in log scale):

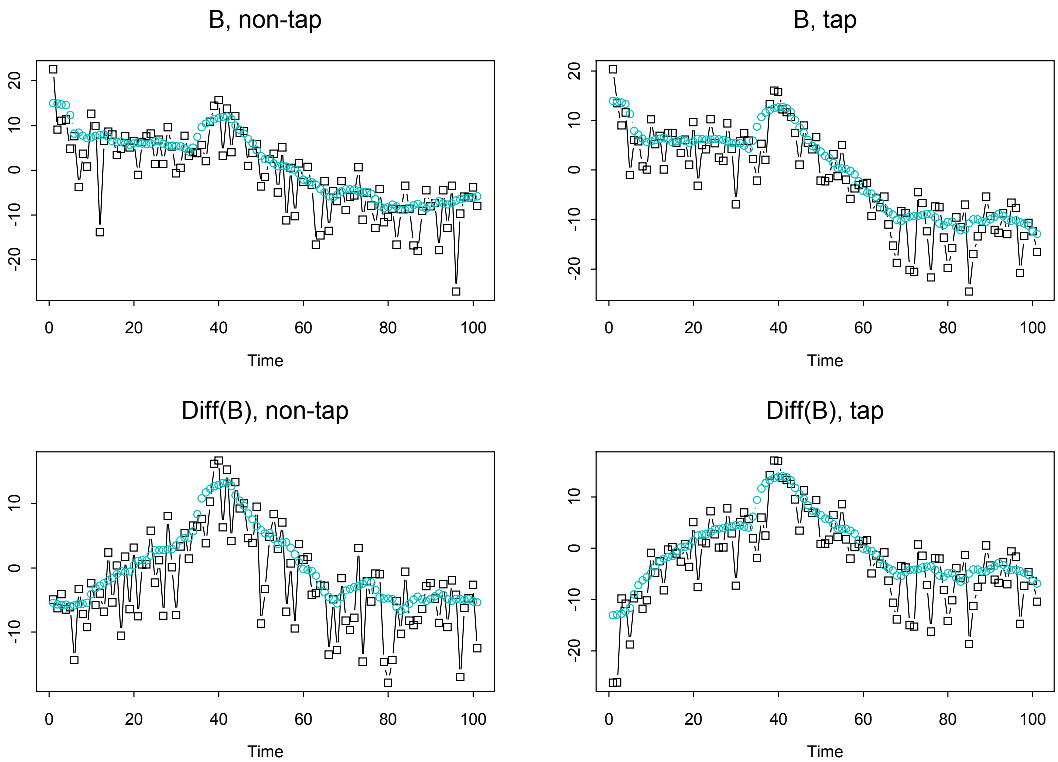
a) [A/B/C, non-tap] the untapered unsmoothed and an untapered smoothed periodogram of the data

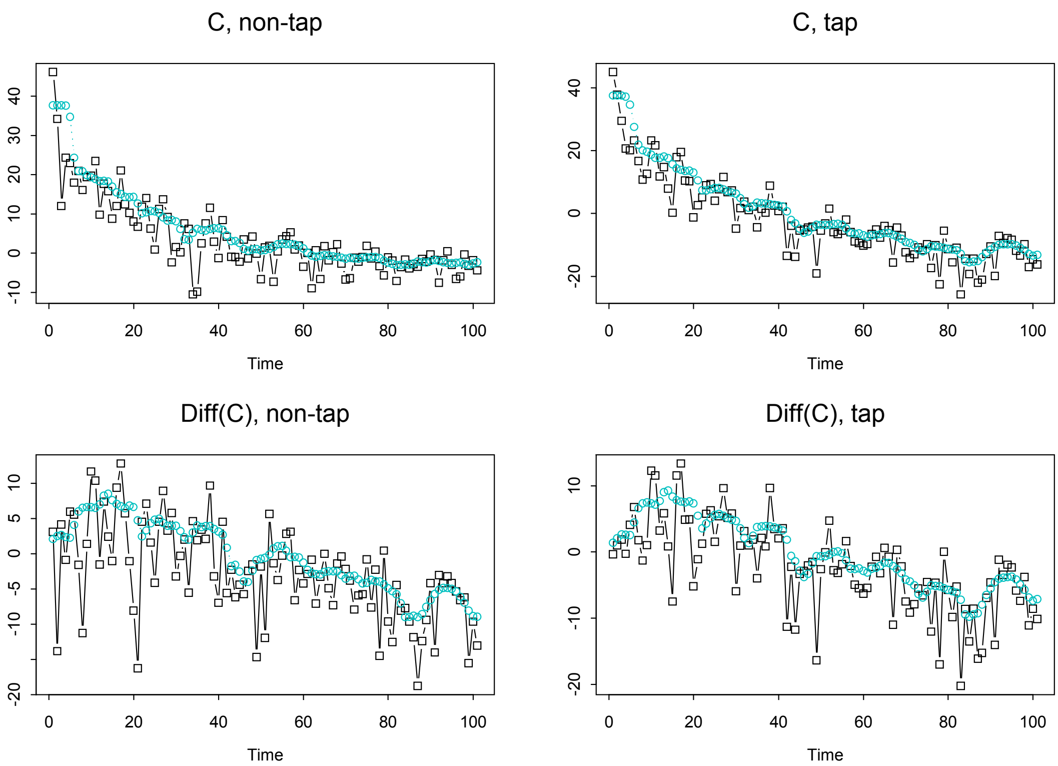
b) [A/B/C, tap] the 30% tapered unsmoothed and a 30% tapered smoothed periodogram of the data.

c) and d) [Diff(A/B/C), non-tap] and [Diff(A/B/C), tap], which are similar as above, but calculated on the basis of the first differences of the data $\Delta X_t = X_t - X_{t-1}$ (instead of the data X_t themselves).

Discuss your suspicions about the models which generated the data. Do they come from an ARMA model (if yes, what do you suspect about the roots of it's polynomials), from a random wave, from a Unit root process, from sums of the above, etc? Explain the differences between the tapered and the non-tapered estimators. (You may have more than one guesses about each model)




B, non-tap
B, tap
Diff(B), non-tap
Diff(B), tap
Time

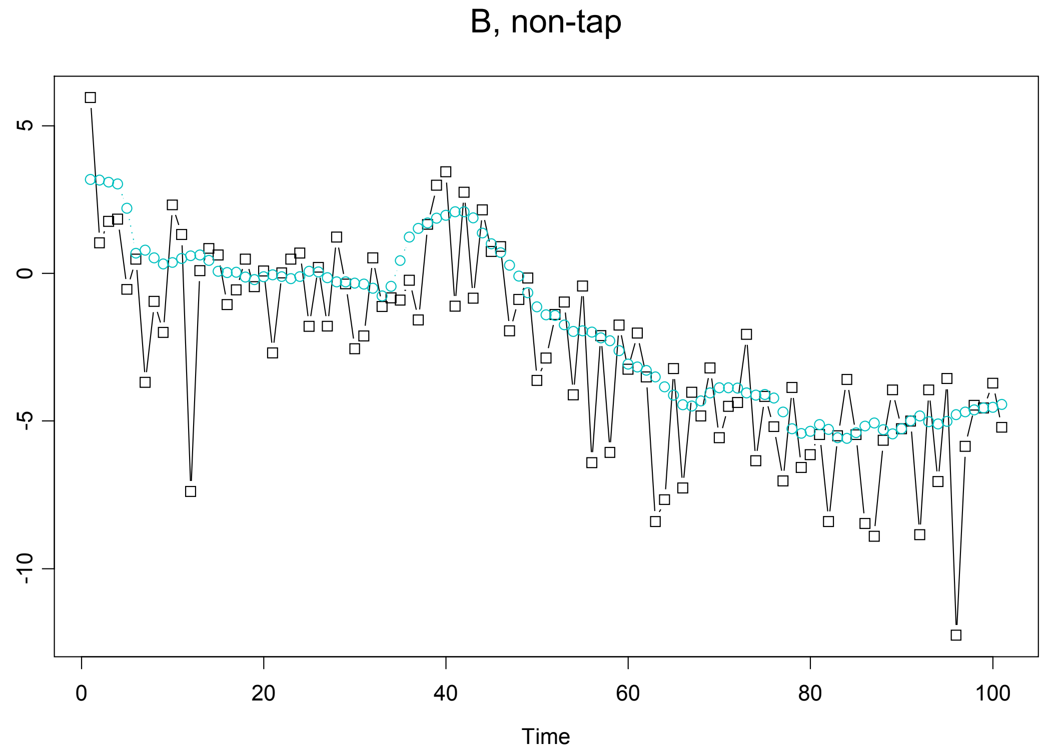

C, non-tap
C, tap
Diff(C), non-tap
Diff(C), tap
Time

4th Question

Below you find the figure of an unsmoothed and of a smoothed periodogram in log-scale (both untapered). They are both based on the same 200 observations. The smoothed periodogram is defined as

$$\hat{f}(\lambda) = \int I(\lambda - \mu)\frac{1}{b}W\left(\frac{\mu}{b}\right)d\mu$$

where b is the bandwidth and W(.) the constant smoothing kernel which is $1/(2\pi)$ in $[-\pi, \pi]$ and 0 outside. Notice that in the calculation of f^(λ) for some λ, only the values of the periodogram I(μ) for μ in [λ - bπ, λ + bπ] are involved. In this example b was chosen in a way that 9 Fourier frequencies lied in [- bπ, bπ].



a) **(0.3 points)** The unsmoothed periodogram seems pretty far away from the smoothed one. Moreover it fluctuates quickly up and down, although it is --theoretically-- a continuous function in λ. Is this the result of too few observations, i.e. would it be smoother and closer to the smoothed periodogram if I had more observations? Explain…

b) **(0.5 points)** What would happen with the bias and the variance of the smoothed periodogram if you increase the number of observations but keep the bandwidth fixed at a specified fraction of π? What would happen to the same quantities if you increase the bandwidth while keeping the number of observations fixed? Would you rather increase or decrease the bandwidth if you have more observations? Why?

c) **(0.7 points)** Calculate 95% asymptotic confidence intervals for the log of the spectrum at a specific λ using the normal approximation. When would you rather use the χ^2 approximation? Why does one calculate confidence intervals for the log spectrum instead of the spectrum itself?

d) **(0.3 points)** What would you need to know (the distribution of which statistic under which hypothesis) if you wanted to calculate simultaneous (valid for all λ at the same time) asymptotic confidence intervals for the log spectrum?

5th Question

(1.4 points) Calculate the expectation of the non-tapered and tapered periodogram of a stationary zero-mean series. On the basis of this calculation, **explain** fully the leakage effect and why tapering reduces it. What is the "disadvantage" of a tapered smoothed periodogram as compared to a non-tapered one, when the leakage effect is small?

6th Question

Assume you observe $X_1,\ldots,X_T$ from a stationary normal time-series with spectrum f.

a) **(2 points)** Show that the variance of the empirical mean tends to 2π f(0)/T when the number of observations tends to infinity. You may assume that the spectrum f is smooth enough for the expectation of the periodogram to tend to the true spectral density when the number of observations tends to infinity.

b) **(1 point)** Give asymptotic confidence intervals for the mean of X_t under the assumption that we have 225 observations which come from a

   b1) White noise process with innovations variance equal to 4.

   b2) An AR(2) process with innovations variance equal to 4 and an autoregressive polynomial which has a root with modulus 1.1 and angle $\pi/4$.

   b3) An ARMA(2,2) process with innovations variance equal to 4, an autoregressive polynomial which has a root with modulus 1.1 and angle $\pi/4$ and a moving average polynomial which has a root with modulus 1.2 and angle $3\pi/4$.

7th Question

(2.3 points) Assume you observe $X_1,\ldots,X_T$ from a stationary series and you want to estimate the spectrum by fitting an autoregressive model of some appropriate order p. For a fixed order p you estimate the autoregressive parameter as the one which best fits the data, i.e. by minimizing --over the parameter value-- the function

"- log Likelihood (data, p, parameter)".

Why is it a bad strategy to select the order p which best fits the data, i.e. which minimizes --over p-- the function

"- log Likelihood (data, p, estimated parameter)"?

If, instead, you select the order p which minimizes --over p-- the function

"- log Likelihood (data, p, estimated parameter) + penalty term",

what property should the penalty term have so that you may hope to select a more "appropriate" order? (What do you want to estimate? What should the penalty term do to the estimator?)