ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

Μεταπτυχιακό πρόγραμμα ειδίκευσης και συμπληρωματικής ειδίκευσης στη Στατιστική

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ΕΞΕΤΑΣΕΙΣ -Μέρος 1ο

Μάθημα : "Προχωρημένες χρονολογικές σειρές"
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1st Question

Let F(λ) be the spectral distribution function of a stationary process $\{X_t\}_{-\infty<t<\infty}$, which has autocovariance function γ(u) and let $\{Z_\lambda\}_{-\pi\le\lambda\le\pi}$ be the orthogonal increment process in the representation $X_t = \int_{-\pi}^{\pi} e^{i\lambda t} dZ(\lambda)$.

a) **(1.5 points)** Which of the following are random variables and which are deterministic quantities:

$$F(\pi/3),\ Z(\pi/3),\ \mathrm{var}[\ Z(\pi/3)- Z(\pi/4)\ ],\ \gamma(3),\ X(10),\ \int_{-\pi}^{\pi} \lambda^2 dZ(\lambda)$$

b) **(1.5 points)** Assume I observe a path of $\{Z_\lambda\}_{-\pi\le\lambda\le\pi}$ and I remark that there is a clear jump (discontinuity) of Z(λ) at λ=π/6. Will F jump as well at λ=π/6? If yes, by how much? Do I know anything about the behavior of γ(u) when u is large? If I observe another path of $\{Z_\lambda\}_{-\pi\le\lambda\le\pi}$ will there probably be again a jump at λ=π/6? A how big one?

2^{nd} Question

Assume I observe $X_1,\dots,X_T$ of a stationary process $\{X_t\}_{-\infty<t<\infty}$ which satisfies the equation: $X_t = 0.99\, X_{t-3} + \varepsilon_t$, where ε_t is white noise with variance 1. I then calculate the moving average Y_t over X_t:

$$Y_t = 1/3\, X_{t-1} + 1/3\, X_t + 1/3\, X_{t+1}\,.$$

a) **(1.5 points)** Calculate the spectral density of X $f_X(\lambda)$ and evaluate it at $\lambda=0$, $2\pi/3$ and π. (Note that f_X hat its peaks at $\lambda=0$ and $2\pi/3$).
b) **(1.5 points)** Calculate the spectral density $f_Y(\lambda)$ of Y and evaluate it at the same frequencies.
c) **(1 point)** How can I see by comparing their spectral densities whether Y will be smoother than X?

($\cos(0)=1$, $\cos(2\pi/3)= -0.5$, $\cos(\pi)= -1$, $\cos(3\pi)= -1$)

3^{rd} Question

Let the spectral density of a stationary process $f(\lambda)$, with $f(\lambda)=1$ for $\lambda \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $f(\lambda)=0$ for $\lambda \in \left[-\pi,-\frac{\pi}{2}\right] \cup \left[\frac{\pi}{2},\pi\right]$.

a) **(1 point)** What do you think about the autocovariance function of the process: will it by $O(1/u)$, $O(1/u^2)$, $\sum|\gamma(u)| < \infty, \sum|\gamma(u)|^2 < \infty$? If you do not remember the exact properties just explain which characteristics of f will be decisive for the behavior of γ and in what sense.
b) **(2 points)** Calculate the autocovariance function of the process.

($d[\sin(\lambda)]/d\lambda = \cos(\lambda)$, $d[\cos(\lambda)]/d\lambda = -\sin(\lambda)$, $\cos(\lambda)= \cos(-\lambda)$, $\sin(\lambda)= -\sin(-\lambda)$)