**ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ**

**ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ**

**Μεταπτυχιακό πρόγραμμα ειδίκευσης και συμπληρωματικής ειδίκευσης στη Στατιστική**

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**ΕΞΕΤΑΣΕΙΣ**

**Μάθημα : "Προχωρημένες χρονολογικές σειρές- time domain"**
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## 1st Question

Assume you observe data $X_1, \ldots, X_T$ which exhibit a linear trend.

- **a.** (0.5 points) Explain the two possibilities of eliminating the trend. How would you chose among them?
- **b.** (0.5 points) Give an example of a model where it is inappropriate to estimate and subtract the trend from the data and where you would rather difference the data once, to eliminate the trend.

## 2nd Question

Consider the following MA(2) time-series model: $X_t = Z_t - Z_{t-1} + 0.5\ Z_{t-2},$ where $\{Z_t\}$ is White noise, with variance 1.

a. (0.7 points) Calculate it's Autocovariance function.

b. (0.6 points) Is this model causal and/or invertible? Which of these two properties tells us whether it is possible to specify an AR(∞) model, the ACVF of which will coincide with the ACVF of the above MA(2)?

c. *(1.2 points) Explain how you could find an AR(p)-model (p finite) with the property that it's ACVF will coincide with the ACVF of the above MA(2) for lags u= 1,.., 10? What p should I choose? (*)

d. (1 point) Consider the model defined by $Y_t = X_t - 2X_{t-1} + X_{t-2}$. Calculate it's ACVF.

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(*) Explain the calculations in enough detail that they could be carried out (for specific values of the quantities involved) on a PC by somebody who is not familiar with the subject. e.g. : set a rxr matrix A= [..] and a vector c=( ). Then calculate $A^{-1}c$, …

## $3^{rd}$ Question

Assume you observe data $X_1, \ldots, X_T$ from a 0-mean AR(2) process $\{X_t\}$ and from these you obtain estimates $\hat{\gamma}(0),\ldots,\hat{\gamma}(T-1)$ of the ACVF in the usual way.

a. (0.4 points) Explain also how you would construct confidence intervals for $\varphi_1$, assuming you have already constructed estimates $\hat{\phi}_1, \hat{\phi}_2, \hat{\sigma}^2$ of the autoregressive parameters $\varphi_1$, $\varphi_2$ and $\sigma^2$. (*)

b. (0.6 points) Explain how you would construct estimates of the PACF $\alpha(u)$, $u = 1,\ldots,20$. (*) How would you recognize in the behavior of the estimated PACF that the model which generated the data is an AR(2)?

c. (1 point) If $\gamma(u)$ denotes the ACVF of the process, which of the following are random variables and which not?

   $X(10)$, $\gamma(5)$, $\varphi_1$, $\hat{\phi}_1$, var($\hat{\phi}_1$), the length of the confidence interval for $\varphi_1$, $E\hat{\gamma}(3)$, $var(\hat{\gamma}(3))$, $\hat{\gamma}(3)$

## $4^{th}$ Question

Assume you are given the values of the parameters $\varphi_1$, $\varphi_2$, $\varphi_3$ and $\sigma^2$ of a zero-mean AR(3) model.

a. *(1 point) Explain how you would calculate its ACVF. (*)

b. *(1 point) Give an argument why the best linear predictor of $X_{T+1}$ based on the last n (n>3) observations is $\varphi_1 X_T + \varphi_2 X_{T-1} + \varphi_3 X_{T-2}$.

## $5^{th}$ Question

Assume you observe data $X_1, \ldots, X_T$ from a stationary process $\{X_t\}$ and from these you obtain estimates $\hat{\gamma}(0),\ldots,\hat{\gamma}(T-1)$ of the ACVF in the usual way. The goal is to test whether $EX_t=0$, based on the sample mean of the data. Researcher A assumes $\{X_t\}$ is White noise. Researcher B assumes $\{X_t\}$ is an AR(10).

a. (0.6 points) Explain why the two researchers will eventually arrive at different conclusions.

b. *(0.3 points) What is the disadvantage of B as compared to A, if A is right is his assumption?(**)

c. *(0.3 points) What is the disadvantage of A as compared to B, if B is right is his assumption?(**)

d. *(0.3 points) What will happen if $\{X_t\}$ is an MA(1) with $\theta>0$? (**)

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(*) Explain the calculations in enough detail that they could be carried out (for specific values of the quantities involved) on a PC by somebody who is not familiar with the subject. e.g. : set a 2x2 matrix A= [..] and a vector c=( ). Then calculate $A^{-1}c$, …

(**) Who is (approximately) controlling the size of the test? Who has better power properties (if both are controlling the size)?

## Some formulas and facts you might need:

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For an MA(q) with parameters $\theta_1,\ldots,\theta_q$, $\sigma^2$ the ACVF is given by

$$\gamma(u) = \sigma^2 \sum_{j=0}^{q-|u|} \vartheta_j \vartheta_{j+|u|}$$

for $|u| \leq q$ (setting $\theta_0=1$).

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For an AR(p) model we have for large T:

$$\mathbf{var}\left(\hat{\phi}_{n,n}\right) = T^{-1}$$

if n>p.

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For an AR(p) model with parameters $\varphi_1,\ldots,\varphi_p$, $\sigma^2$ the Yule-Walker equations are:

$$\gamma(\kappa) - \phi_1 \gamma(\kappa-1) - \ldots - \phi_p \gamma(\kappa - p) = \begin{cases} \sigma^2, \text{if } \kappa = 0 \\ 0, \text{else} \end{cases}$$

for $\kappa = 0, 1, 2, \ldots$ arbitrary.

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